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Computation of Total Kulli-Basava Indices on a Few Specialized Families of Graphs

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Abstract: The index discussed in this paper is a non-negative integer derived from a graph, defined as the sum of degrees of all edges incident to a given vertex. We extend the concept of the Total Kulli-Basava index to more complex graph structures and derive several results relating Total Kulli-Basava index across Wheel graph, Gear graph, Helm graph, combined Helm Wheel graph, combined Helm Gear graph and Extended Helm Gear graph.

Keywords: Combined Helm Wheel graph, Combined Helm Gear graph and Extended Helm Gear graph, Total Kulli-Basava index.
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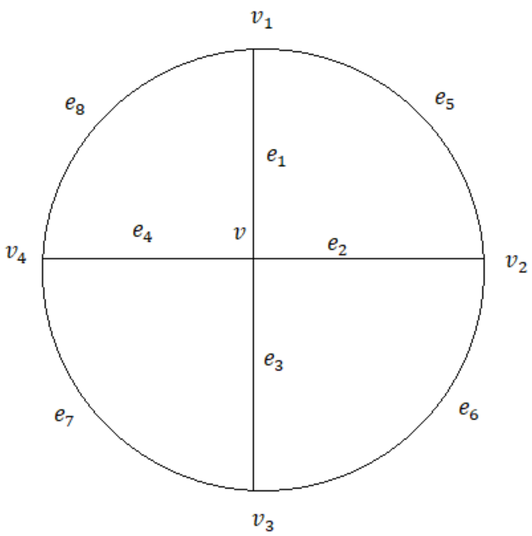
1. Introduction

Let G be a simple connected finite graph. $V(G)$ and $E(G)$ denotes the vertex set and edge set. $S_e(u)$ denote the sum of the degrees of all edges incident to a vertex u.

Consider the wheel graph W_4 with 5 vertices and 8 edges.

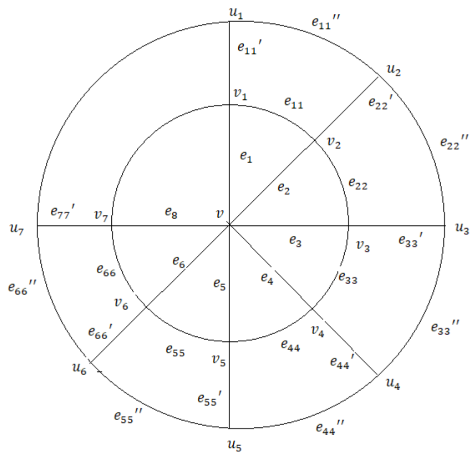


Then $S_e(v)$=20 and $S_e(v_i) = 13$
In general $S_e(v)$ of W_n is $n(n+1)$ and $S_e(v_i) = n + 9$
The Total Kulli-Basava index and First Kulli-Basava index of a graph, are defined as $TKB(G) = \sum_{u \in V(G)} S_e(u)$ and $KB_1^*(G) = \sum_{u \in V(G)} S_e(u)^2$ respectively.The general multiplicative Kulli-Basava index of a graph is $KB^a II(G) = \prod_{u \in V(G)} S_e(u)^2$

In this paper, we compute explicit formula for computing Total Kulli-Basava index and First Kulli- Basava index and general multiplicative Kulli-Basava index of combined Helm wheel graph, combined Helm gear graph and extended Helm gear wheel graph.

2. Combined Helm wheel graph W_n

Helm wheel graph is obtained from Helm graph by attaching edges to complete the outer rim.



Clearly HW_n has $2n + 1$ vertices and $4n$ edges**.** HW_n graph has three types of vertices.

$S_e(u)\backslash u \in V(\mathrm{HW_n})$	$n(n+1)$	$n+19$	13
Number of Vertices	1	n	n

Table 1: Vertex partition of HW_n

Theorem 2.1: Let HW_n be a Helm wheel graph with $2n + 1$ vertices and 4 $n$edges.Then general multiplicative Kulli-Basava index of $\mathrm{HW_n}$ is $KB^a II(\mathrm{HW_n}) = 13^{na} n^a (n+2)^a (n+19)^{na}$.

Proof: $KB^a II(\mathrm{HW_n}) = \prod_{u \in V(\boldsymbol{HW_n})} S_e(u)^a$

$$= (n(n+2))^a (n+19)^{na} 13^{na}$$
$$= 13^{na} n^a (n+2)^a (n+19)^{na}$$

Theorem 2.2: HW_n be a Helm wheel graph with $2n + 1$ vertices and $4n$ edges.Then total Kulli-Basava index of $\mathrm{HW_n}$ is $2n(n + 17)$

Proof: $TKB(HW_n) = \sum_{u \in V(\boldsymbol{HW_n})} S_e(u)$

$$= n(n+2) + n(n+19) + 13n$$
$$= 2n(n+17)$$

Theorem 2.3: The modified First Kulli-Basava index of HW_n is $KB_1^*(\mathrm{HW_n}) = n^4 + 5n^3 + 42n^2 + 530n$

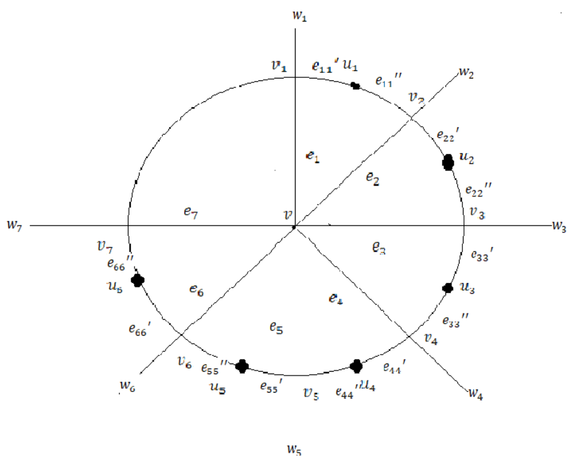
Proof: $KB_1^*(\mathrm{HW_n}) = \sum_{u \in V(\mathrm{HW_n})} S_e(u)^2$

$$= n^2(n+2)^2 + n(n+19)^2 + n13^2$$
$$= n^4 + 5n^3 + 42n^2 + 530n$$

.

3. Combined Helm Gear Graph ($\mathrm{HG_n}$)

Helm Gear Graph is a graph obtained from Helm graph by attaching vertices in between 2 adjacent edges in the rim. Clearly $\mathrm{HG_n}$ has $3n + 1$ vertices and $4n$ edges. $\mathrm{HG_n}$ graph has 4 types of vertices.



$S_e(u)\backslash u \in V(\mathrm{HG_n})$	$n(n+2)$	$n+13$	8	3
Number of Vertices	1	n	n	n

Table 2: Vertex partition of HG_n

Theorem 3.1: Let $\mathrm{HG_n}$be a Helm Gear Graph with $3n+1$ vertices and $4n$ edges.Then general multiplicative Kulli-Basava index of $\mathrm{HG_n}$ is $KB^aII(\mathrm{HG_n}) = 24^{na}n^a(n+2)^a(n+13)^{na}$.

Proof: $KB^aII(\mathrm{HG_n}) = \prod_{u\in V(\boldsymbol{HG_n})} S_e(u)^a$

$$= (n(n+2))^a(n+13)^{na}8^{na}3^{na}$$

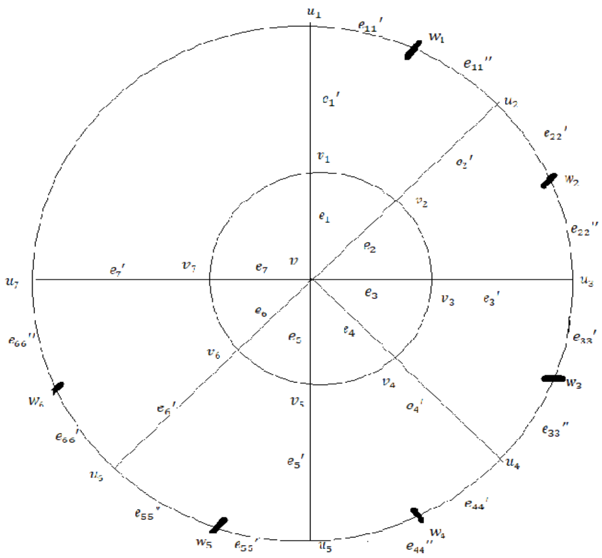
$$=24^{na}n^a(n+2)^a(n+13)^{na}$$

Result 3.1: Total Kulli-Basava index of $\mathrm{HG_n}$ is $2n(n+13)$

Result 3.2: The modified first Kulli-Basava index of HG_n is $KB_1^*(\mathrm{HG_n}) = n^4+5n^3+30n^2+262n$

4. Extended Helm Gear Graph($\boldsymbol{HG_n^*}$)

Extended Helm Gear Graph(HG_n^*) is obtained from Helm Graph by attaching one vertex and two edges to complete the outer rim. So HG_n^* graph has $3n+1$ vertices and $5n$ edges.



$Se(u)\backslash u \in V(HG_n^*)$	$n(n+2)$	$n+19$	11	6
Number of Vertices	1	n	n	n

Table 3: Vertex partition of HG_n^*

Result 4.1: The general multiplicative Kulli-Basava index of HG_n^* is $66^{na}n^a(n+2)^a(n+19)^{na}$.

Result 4.2: The Total Kulli-Basava index of HG_n^* is $2n(n+19)$

Theorem 4.1: The Total Kulli-Basava index of a graph is always multiple of four.

Proof: Since the sum of degrees of vertices in a graph is even and in $S_e(v)$, degree of a vertices of any graph is repeated 2 times.

Theorem 4.2: Number of vertices of odd $S_e(v)$ of a graph is even.

Proof: The Total Kulli-Basava index of a graph is always multiple of four, that is $\boldsymbol{TKB}$ of a graph is even. Hence number of vertices of odd $S_e(v)$ of a graph must be even.

5. Results connecting $TKB's$ of Wheel graph(W_n),Gear graph(G_n),Helm graph(H_n),(HW_n), (HG_n) and HG_n^*

1)$TKB(HG_n^*) - TKB(H_n) = \{TKB(HG_n^*) - TKB(HG_n)\} + \{TKB(HG_n^*) - TKB(HW_n)$
2)$TKB(HG_n^*) - TKB(G_n) = 2\{TKB(HG_n^*) - TKB(HG_n)\}$
3)$TKB(HG_n^*) - TKB(W_n) = \{TKB(HG_n^*) - TKB(HW_n)\} + \{TKB(HG_n^*) - TKB(G_n)$
4)$TKB(HG_n) - TKB(H_n) = \{TKB(HG_n) - TKB(W_n)\} + \{TKB(HG_n) - TKB(HW_n)\}$
5)$TKB(HG_n) - TKB(G_n) = \{TKB(HG_n) - TKB(W_n)\} - \{TKB(HG_n) - TKB(H_n)\}$
6)$TKB(HG_n) - TKB(W_n) = \{TKB(HG_n) - TKB(H_n)\} + \{TKB(HG_n) - TKB(G_n)\}$
7)$TKB(HW_n) - TKB(H_n) = \frac{1}{2}\{TKB(HW_n) - TKB(W_n)\}$
8)$TKB(HW_n) - TKB(G_n) = \{TKB(HW_n) - TKB(HG_n)\} + \{TKB(HW_n) - TKB(H_n)$
9)$TKB(HW_n) - TKB(W_n) = 2\{TKB(HW_n) - TKB(H_n)\}$

6. Definition of $T_v^{'}KB$ of the graph

Let v be the central vertex of $W_n, G_n, H_n, HW_n, HG_n, HG_n^*$. Define the sum $T_v^{'}KB(G) = \sum_{u \in V(G\setminus v)} S_e(u)$, where G is one of the above graphs.

Then
1) $T_v^{'}KB\,(HG_n^*)$ - $T_v^{'}KB\,(HG_n) = 12n = TKB\ (HG_n^*) - TKB\ (HG_n)$
2) $T_v^{'}KB\,(HW_n)$ - $T_v^{'}KB\,(H_n) = 12n = TKB\ (HW_n) - TKB\ (H_n)$
3) $T_v^{'}KB\,(HG_n^*)$ - $T_v^{'}KB\,(HW_n) = 12n = TKB\ (HG_n^*) - TKB\ (HW_n)$
4) $T_v^{'}KB\,(HG_n)$ - $T_v^{'}KB\,(H_n) = 4n = TKB\ (HG_n) - TKB\ (H_n)$
5) $T_v^{'}KB\,(G_n)$ - $T_v^{'}KB\,(W_n) = 4n = TKB\ (G_n) - TKB\ (W_n)$

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