Review and Recent History of the Fractals Research in Bulgaria

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Abstract: The review is presenting the history and development of the recent fractals' research in Bulgaria during the last decades. The aim of this study is to make more familiar the topic in front of the international fractal's society. It is focused on the entire list of publications related to this research attached as Appendix. Additionally several examples, illustrations and comments are presented just to give a short view to the results of these investigations. Everyone who needs to be more familiar with the methods, data and results can use the bibliography to find more information about the achievements on this topic. The structure is following simple rules– initiation, maturity stage and recently presented papers. Tables show fractal dimensions determined for different examples from the fields of geodynamics on the Earth, some planets and satellites of the Solar system, applications to the seismic hazard assessment, topography and digital elevation models and their relationships with geophysical fields, etc. There is also an example of the fractal analysis of visual art pictures, etc. The conclusions confirm the effective results, potential to progressive development, needs to fulfill gaps and future vision on the development especially focused to the younger generation of investigators.
Keywords: Bulgaria, Fractals, Research,Review.

1. Introduction

This review is a personal view about the last decade's research and development related to fractals and fractal analysis in Bulgaria. It deals with the development and application of the fractal analysis. Bulgaria is a small country, not familiar to the fractal society due to the limited number of specialists cooperating and developing this very useful tool for solving different tasks in broadband subjects and applications in many new emerging fields of knowledge. The specialists using fractal analysis and its results in Bulgaria are in general geophysicists, geomorphologists, space research specialists, geodynamics and planetary dynamics experts, artists, etc. The aim of this Review is to make the achievements of investigators in the field of fractal analysis more familiar to the professionals all over the world, presenting shortly the results and new knowledge extracted by the application of the fractal investigations. The review is structured in several chapters: Introduction, Inspiration and first steps, Methods and algorithms, Investigations and selected results (with selected illustrations and comments), Development and evolution, Gaps and future challenges, Conclusions, Appendix – Bibliography.

2. Inspiration and first steps

This is a very brief synthesis of the results and achievements of the fractal analysis in Bulgaria, using different methods according the targets and tasks to solve by specialists in broadband field of knowledge. The focus is on the popularization of investigations done during the last decades and for this a list of publications is presented as Appendix. This approach formulated and presented the use of fractal analysis as effective tool for description, visualization and solutions of different natural phenomena's investigations related to different objects – from the geodynamics to the space remote sensing imaginary, from seismotectonic models to the geometry of coastal tsunami prone areas, from digital elevation models of the planets to the structure of new discoveries, even from the art pictures to the unusual solutions about "parallel realities".

The start was inspired by the book of B Mandelbrot "The Fractal Geometry of Nature" [1] – a famous view to the nature, visible in everyday life by everyone but not recognized as a common property of any no regular object for very long time of the civilized humanity. It is so amazing that during the whole history of the development of mankind, people do not recognize such obvious and easy to see property of the nature. In this sense the fractal analysis is a late discovery of the human society and the main price has to go to Mandelbrot. The first attempt in Bulgaria to use fractal analysis was made in 1997 to describe the coast line and bathymetry of the Black see (the analogy with the "West Coast of Britain" is rather clear) with regards of the tsunami hazard to the coastal areas. The world practice related fractal and fragmentation to Geophysics [2] and application to Earth's sciences [3].

Then following the international practice and considering the effectiveness of the method extends the applications to the broader direction of seismotectonics and seismic hazard modeling, geodynamics and so on and so on. Relatively easy way to find (even graphically) the fractal dimension (a unique number measuring nonlinearity) was used like a magic tool to solve many and different problems. Even more the results obtained show that the researcher has a new tool for investigation, giving deeper insight, showing new properties and giving a lot of positive results. This approach was a starting point of extended research and investigations in very broadband fields of sciences in Bulgaria.

The development of the theory after the Mandelbrot's book created a lot of publications in different variety of applications and reviews' samples [for example [4] - Fractal Geometry and Spatial Phenomena. A Bibliography by Mark MacLennan, A. Stewart Fotheringham, and Michael Batty (1991)] was the second inspiration idea. The idea is

very good and gave a broader view but is limited only on the presentation of the bibliography parameters of published works. Our goal is wider – not only to give a bibliography list of publications, but also to present main achievements of the investigations related to fractals and fractal analysis in Bulgaria.

3. Methods and algorithms

A short review of the recent methods and software for fractal dimension calculation shows only the tools used in different research projects to formulate and solve the tasks, regarding the aims and scope of the presented achievements. Some figures are also included just to illustrate the main results of the investigations.

3.1 "Number–Size" Method

This is probably the simplest method for calculation the fractal dimension. It presents the number of the sizes of different objects – points, areas, volumes – most frequently in semi-log scale. The fractal dimension is easy to obtain just dividing the presented parameter in largest interval to the number of cases observed. This method is frequently used when fragmented objects are studied.

3.2 Anti-Aliasing Algorithm

Used to study structures with chaotic geometry, surface fragmented objects, etc.

3.3 "Sandbox" Method

The fractal dimension D is determined from the slope of the linear part of the graph $logN(r)=Dlogr+C$. The number of points $N(r)$ and the radius r are presented on a logarithmic scale.

3.4 Divider Method

This method measures the length of a curve using different "step" sizes d. As d decreases, the measured length $L(d)$ increases, allowing D to be calculated from the relationship $L(d) \sim d(1-D)$.

3.5 Variogram Method

It uses spatial autocorrelation by the variogram. The fractal dimension is estimated as $D=E+1-H$ (E – Euclidean dimension, H – Hurst parameter).

In the Literature review can be find more methods, like MULTIFRACTAL ANALYSIS, WAVELET ANALYSIS, FURIER SPECTRAL METHOD, THE BOX-COUNTING METHOD, etc.

4. Investigations and Selected Results

In general the synthesis of the main areas of the fractal analysis applications in Bulgaria might be structured on typology of the publications:

4.1 Application of the fractal dimensions and analysis to the global and regional geodynamics

Due to the fragmentation in many cases of the global geodynamics, the fractal analysis is a suitable tool for research and interpretations. Starting from the seismotectonic models of Bulgaria, Balkans and Mediterranean (according Jimenecetal, [5] - Fig.1.) several fractal dimensions were determinate (TABLE 1) regarding the geometry of seismic active sources, density of epicenters, seismic activity, repeatability, magnitude and depth distributions of earthquakes and some other parameters characteristic for the different seismic regions in Mediterranean and Europe. The linear and areal elements of the geometrically formalized seismic sources have been calculated. The seismotectonic models of the above mentioned areas were proved as fractals for the first time. Determination of the fractal dimensions of these not homogeneous areas of seismic hazard helps to calibrate the hazard assessment and to add a new tool to the seismic risk calculations there.

Research of the recent Earth's crust surface fragmentation in Bulgaria was another aspect demonstrating the power of the fractal analysis to describe the near surface and deeper faulting for the geodynamic activity observed in Bulgaria [6].
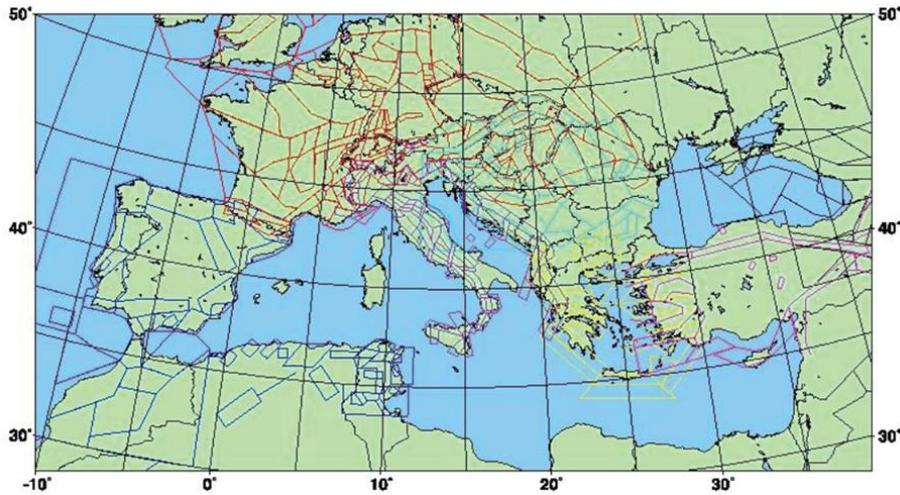
Figure 1:Seismotectonic model of the Euro-Mediterranean area. Seismic sources are presented as no regular polygons (according Jimenes et al,[7].)

Table 1: Fractal dimensions of the linear (DL) and surface (DS) elements for all seismogenic provinces of the Euro-Mediterranean seismotectonic model.		
Zone	$D(L)$	$D(S)$
AD (magenta)	2,71	1.67
CWE (brown)	1,12	0.41
PWA (dark blue)	1,18	0.24
GR (yellow)	0.94	0.40
BG NB (light blue)	1.20	0.25
All zones	1.23	0.38

Legend (Seismic Regions): AD-Adriatic, CWE – Central and West Europe, PWA – Pyrenees and West Africa, GR – Greece, BG NB – Bulgaria and North Balkans

More than 20 publications are dealing with this topic. The summary paper compilation is published in 2010 when the results about nonlinearities and fractal properties of the Euro-Mediterranean seismotectonic model are presented.

Larger global view was proved about the global plate tectonic (PT) model (Bird, 2003 – PT [7]) as a fractal. For the first time in history of the global Earth geodynamics the fractal properties of the plate tectonic paradigm are determined considering sizes of the variety of elements of the Plate Tectonics: tectonic plates, subduction zones, orogens, collision and rift zones together with the transform faults (these are practically all elements of the Earth's plate tectonics). They have been investigated from fractal point of view. The results prove that almost all elements demonstrate fractal properties - Fig.2. (The fractal dimensions are calculated and presented on TABLE 2). The weirdest fact is that all elements described earlier are fractals with only one exception – the transform faults-(Fig.3) [8].
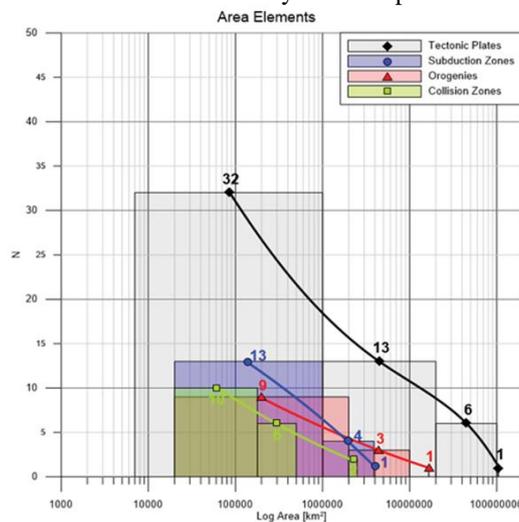

Figure 2: Areas of main tectonic plates (black), orogens (red), subduction zones (blue) and collision zones (gren) on semi-log graph.

Table 2: Fractal dimensions of the Plate Tectonic (PT) elements

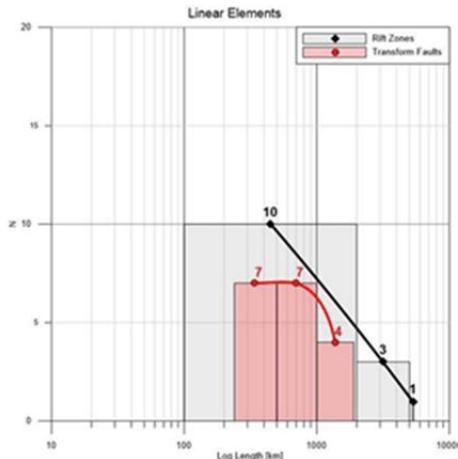| Elements of PT	Linear	Areal	Notes
Tectonic plates		3.01	Highfragmentation
Rift zones	1.81		Lowestfragmentation
Orogens		5.32	Highestfragmentation
Subduction zones		3.07	Highfragmentation
Major Collision zones		3.28	Highfragmentation
Major transform faults	N/A		Seems not tobe a fractal

Figure 3: Linear sizes of rift zones and transform faults (according [7]) on semi-log graph. The transform faults are not fractal even visually recognized.

The interpretation of this fact is related with the idea that the transformation (demonstrated by global faults) are still in earlier stage of their development and still didn't reach the maturity to be considered as fractal.

The fractal shape of the Maldives archipelago was proved by many publications and a summary paper [9] (Fig.4.) helped a lot for the suggestion of a unique hypothesis about the creation of the Maldives [10].
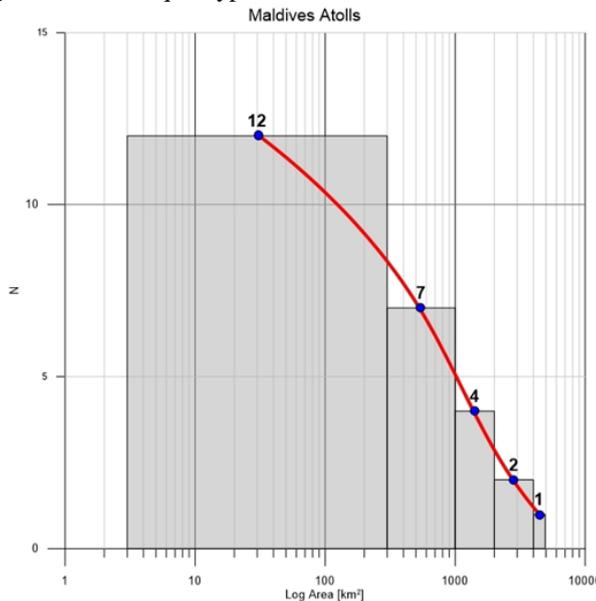

Figure 4: Surface areas of main atolls of Maldives archipelago, showing clear fractal view on semi-log graph.

4.2 Fractal research of the earth's morphostructures and natural geophysical fields

Several years of investigations have been targeted to the morphology of the surface Earth structures for the Balkans. Attempts to relate these topography structures with the different natural geophysical fields (mostly gravity anomalies – Bouguer, free-air, isostasy) have been explored [11] (Fig.5.and Fig.6.)
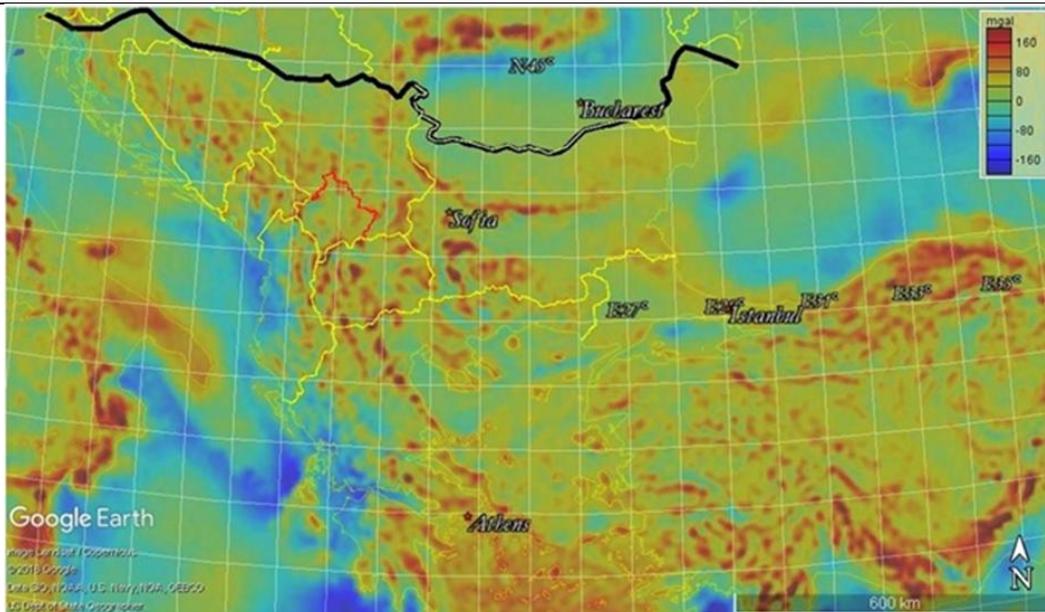
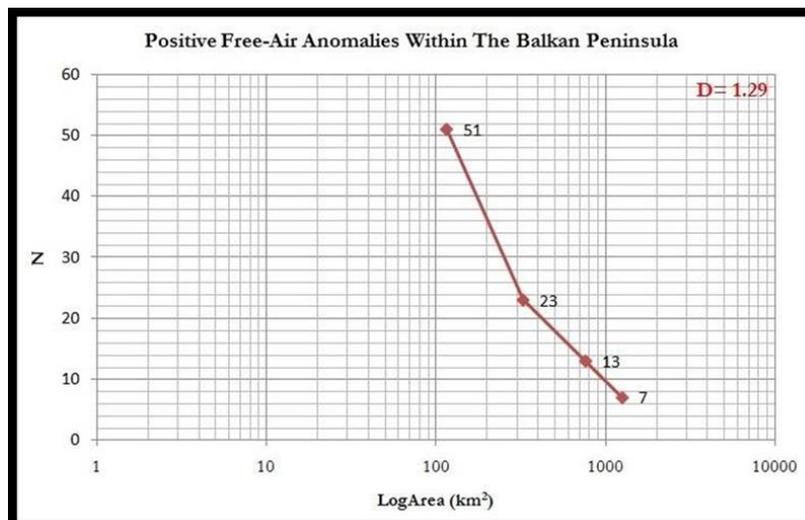Figure 5: Free-air gravity field on the Balkans (according [11])

Figure 6: Fractal dimension determined from the semi-log graph for the positive free-air gravity anomalies on the Balkans (acc.[11])

4.3 Fractal approach to the Solar system, planetary dynamic properties and satellites

The increased experience of many scholars dealing with the fractal analysis on the Erath, leads to a new era started with the investigation of other space bodies and many of their parameters by the fractal analysis. The fractality of different elements and availability of data from the space missions give the possibility to perform the methods of fractal analysis on the other space bodies – Moon, Mars, and finally entire Solar system [12].

The investigations were focused on the topography and the relations to the crater formation, gravity field and seismicity of the space bodies. The Digital elevation models (DEM) of the surfaces of these bodies with enough high resolution presented a unique possibility to search the correlations between the different parameters, their relationships and their fractal dimensions as indicator of fragmentation of these elements (surface structures – lowlands and mountains, faults, craters – volcanic and impact on the Moon, former river beds on Mars, gravity field of Mercury, etc. have been investigated and results published in variety of papers.

The world tendency of increased interest to the study of these elements reflected also in Bulgarian research. Correlations between fractal dimension of many surface parameters (for example DEM) and the measured natural fields predominantly gravity anomalies, as well as the morphology and the differences between North and South hemispheres of Moon and Mercury have been investigated [13] – Fig 7.and TABLE 3.
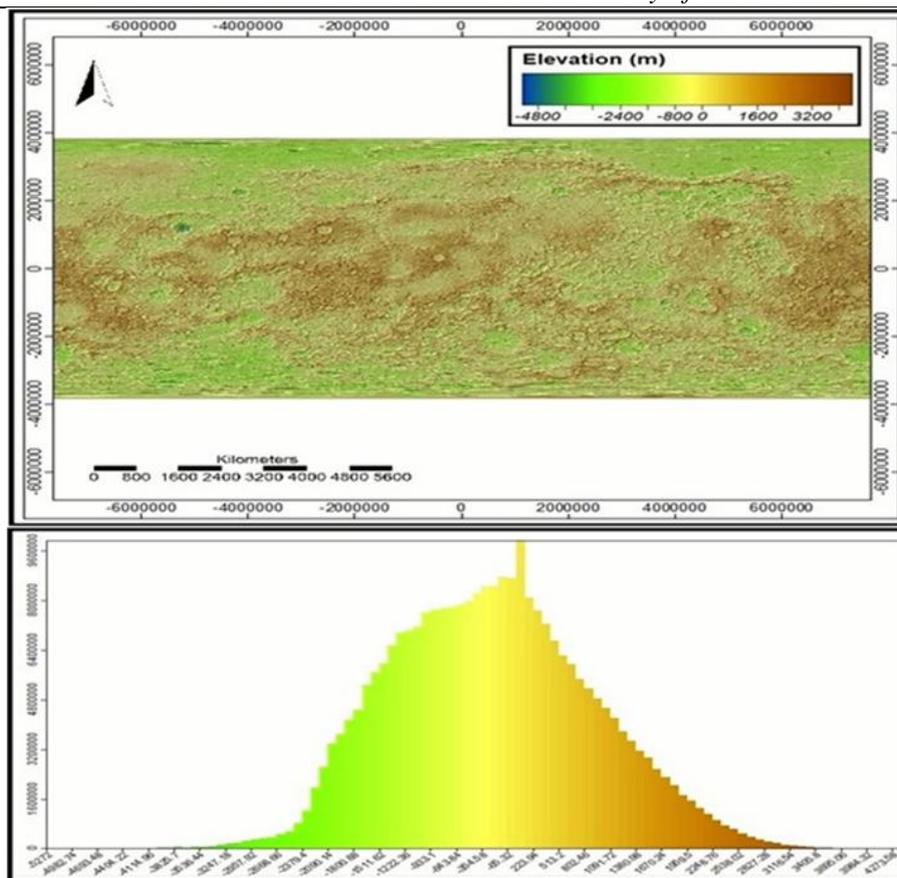
Figure 7: Mercury DEM (up) and fractal dimensions (down) distribution (acc. [13]).

Table 3: Fractal dimensions of the Free-Air gravity anomalies and DEM for Mercury's hemispheres and correlation coefficients.

	Free-Air Gravity (FAG)					
	Northern hemisphere			Southern hemisphere		
	All values	Positive	Negative	All values	Positive	Negative
FD min	2,10	2,01	2,01	2,13	2,01	2,01
FD max	2,56	2,38	2,41	2,64	2,40	2,44
R^2	0,908	0,968	0,967	0,975	0,994	0,990
	Digital Elevation Model (DEM)					
	Northern hemisphere			Southern hemisphere		
	All values	Positive	Negative	All values	Positive	Negative
FD min	2,01	2,0	2,01	2,01	2,0	2,01
FD max	2,64	2,45	2,58	2,58	2,47	2,54
R^2	0,942	0,903	0,940	0,857	0,863	0,910

The results of these investigations put on discussion the most important question – Is the fractal fragmentation result of the Genesis of the space objects or it is a result of the "geodynamic" space-time evolution of the solid terrestrial space bodies? Up to now it seems that the fractality in the Solar system's terrestrial objects is due to the influence of both factors – genesis and evolution - sometimes prevailing of one or another.

4.4 Fractal investigations of the seismic activity, seismotectonic models and seismic hazards assessment
Going deeper in details of the seismotectonic models represented as set of not regular geometric objects, the fractal analysis determined fractal dimension used later for the seismic hazard assessment. For example – the shallow earthquake sources of the Balkan Peninsula have been presented as no regular set of polygons – fig.8. Further on this view was used in the seismic hazard and risk assessment.
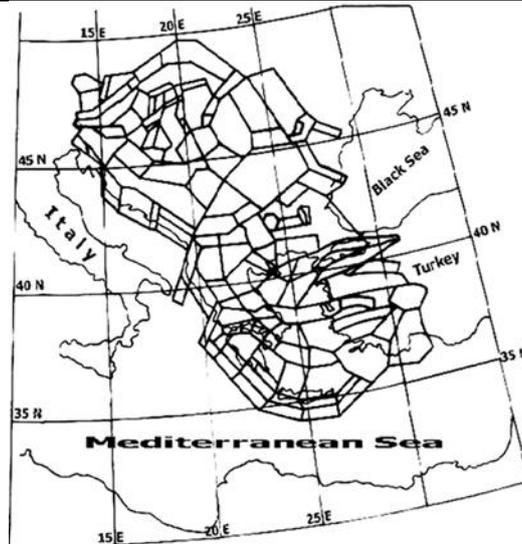
Figure 8:Seismotectonic model of the Balkans – proved fractal for seismic hazard assessment.

4.5 Other Applications of fractal analysis in (visual arts, physical discoveries, etc.)

Using BOX-COUNTING method a fractal analysis was performed on visual pictures of different origin, but created by a single author. The methodology of visualization is rather specific and completely different from previous investigations in the modern visual art [14]. The mentioned paper triggered great interest among researchers and many papers were published. The importance of Bulgarian research is due to the preparation of the initial picture to fractal analysis using multiplication, mirroring and consecutive fractal analysis starting from the single and then several times multiplicationed pictures. The results of this research is presented just like curious fact for fractal analysis of visual art in Bulgaria [15] - Fig 9 to Fig. 12.and TABLE 4.
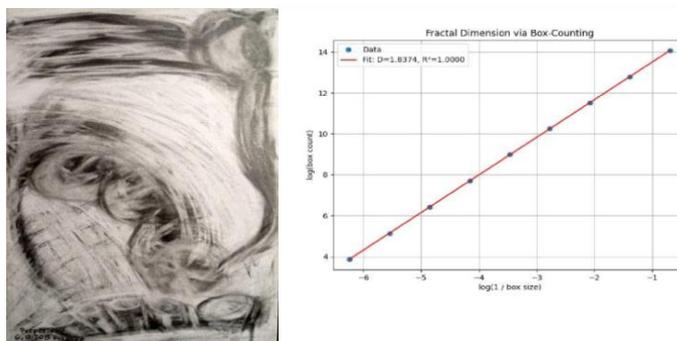

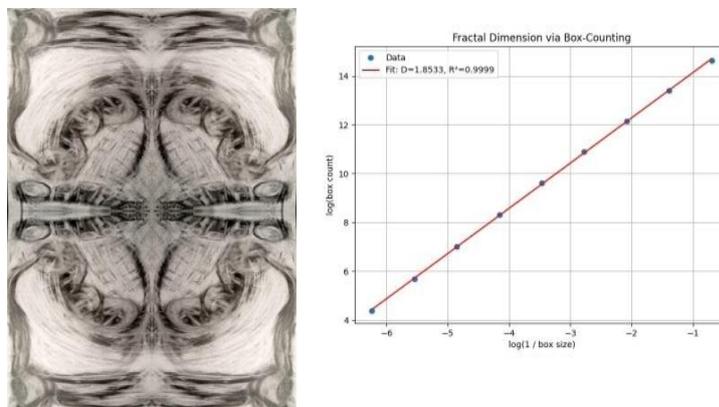Figure 9: Original Image induced by Perperikon Archaeological site and its fractal graph.

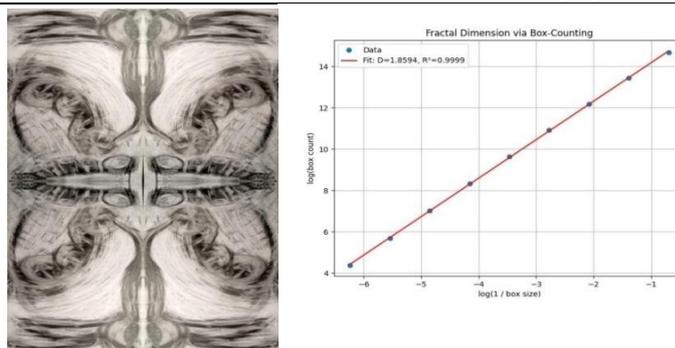Figure 10: Multiplication of picture four times with the fractal graph.

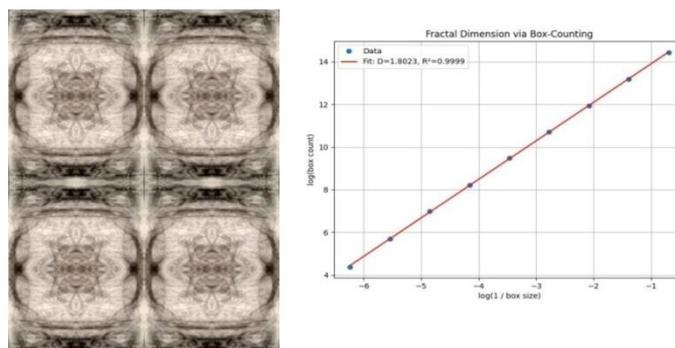Figure11: Mirrored four times multiplicated picture with the fractal graph

Figure 12: Mirrored 4x4 multiplicated pictures and the fractal graph.

Table 4: Perperikon initial picture fractal dimension FD_{1o} and correlation coefficient R_{1o} and FD_{1-3} and R_{1-3} for the multiplicated pictures.

Case	FD_{1o}	R_{1o}	FD_1	R_1	FD_2	R_2	FD_3	R_3
Perper	1.8374	1.0000	1.8594	0.9999	1.8533	0.9999	1.8023	0.9999

The table 4 presented the extraction of determined parameters and their comparison shows small deviations in the fractal dimension (from 1.80 to 1.86) which is in the range of 0.04 and the very strong correlation R – 0.999 to 1.000.

The very similar results of the fractal dimensions of completely different pictures investigated and the very high correlation gave the possibility to suggest a hypothesis that the different pictures created at different places and during different time intervals were inspired to the artist by the same source. This fact was never before suggested as a hypothesis.

5. Development and Evolution

The stages of development of the fractals' investigations in Bulgaria are conditionally separated timely and typologically. Topics of research, Main results and Bibliography sources are arranged to follow by easiest way the "initial", "flourish" and "recent" stages of this development - TABLE 5.

Table 5: Development stages and evolution – research topics, main results and bibliography sources

Stages (years)	Topics of research	Main results	Sources[Bibliography]
Initial (1997-2017)	Seismotectonics, Tsunami hazard, Faults' fragmentation	Coastal fractal of Black Sea, Seismotectonic models of Balkans and Euro-Mediterranean	[1]- [8] [GD], [ST], [SSH], [Ts].
Flourish (2018-2020)	Geodynamics, Natural geophysical fields, Seismicity and point distributions, Geomorphology, Natural Hazards -Seismic (and tsunami).	Global Plate Tectonics, Morphology analysis, Correlations (DEM-Geophysical fields), Models of seismicity and hazards	[9]-[27] [GM], [S], [T], [SSH],[CNFM], [PDM]
Recent (2021-2025)	Space research – Moon and Mars topography, study of DEM and gravity field, Planetary and satellites "geodynamics"	Correlations (DEM and Geophysical fields), morphology of planet's surface, Planetary dynamics evolution.	[29]-[34] [GD,[PDM],[T],[S]
Miscellaneous	Visual art, Physics	Fractality proved	[28],[35]

Legend of scientific disciplines:

Geodynamics [GD], Seismicity [S], Tsunamis [Ts], Seismotectonics [ST], Seismicity and seismic hazard [SSH}, Topography [T], Planetary Dynamics and Morphology [PDM], Correlations of natural fields and morphology [CNFM].

Three books were published ([36]-[38] from the Bibliography) as synthesis of many earlier publications.

These topics are separated for easier presentation for the reader and are focused to indicate the disciplines where the analysis is performed. The indices are attributed to each publication at the end of the citation. The references about other topics related to the applications are numbered at the Reference at the end of the review.

The initial stadia covered ten years' time interval – 1997-2017 and 9 publications in total (8 papers and a book). It was targeted in general to the seismotectonic models and seismic hazard assessment and their fractal properties, tsunami research and Global geodynamics.

The "flourish" range covered 3 years 2018-2020 and had 22 papers and two books. Earth's geodynamics, morphostructural analysis, correlations between DEM and geophysical fields on Earth and space terrestrial bodies, seismicity and natural hazards are the main characteristics of this time interval.

The last recent years from 2021 to 2025 are focused mostly on space planetary dynamics, seismicity patterns and some exotic applications: The discovery of Dichev's rings in magnetism and their fractal properties; The fractality of the drawings by R. Kasabova recognized as real fractals with higher correlation coefficients. In total 8 publications are presented and they are a solid fundament of the upcoming new monograph as summary of these investigations and their interpretations and results. In general this analysis concludes that the research is focused to less theory, but to more pragmatics and practical approach, using new developed software available nowadays.

Many of publications are repeating the content and presented to different scientific events (conferences, symposia, etc.) to make more popular our research in the front of different societies – geographers, geophysicists, geodynamic people, geomorphologists, astrophysicists, etc.

6. Gaps and future challenges

In this review we do not discuss the development of the fractal theory and mathematical challenges such as AI, theory of groups and rings and even "parallel realities", etc.

From one side:
1. The small number of scientists using the fractal theory and practice in Bulgaria is the main gap. It needs to be fulfilled by more and younger specialists in these fields.
2. The lack of knowledge is a result that no educational programs include this topic in their curriculum. Even in mathematical specialties, engineering and other natural sciences this topic is not included into educational process.
3. The self-education is not the best way paving the road to this very specific, but very useful tool.
4. Many other applications are not covered: Except in the geology knowledge and planetary sciences, but also in the frame of new materials development, pharmacy, chemistry (with the combination of chiral components), structural mechanics, architecture, visual arts (for identification of authorship), archaeology (for comparative analysis of artifacts and old buildings), probably also in electronic music, the fractal analysis can help to increase efficiency in many scientific and cultural applications.

On another side:
1. The specific topic as fractals needs a specific approach to the data collection.
2. The specially targeted instrumentation of different space missions for the planetary and Earth's remote sensing is better to develop specialized equipment with specific characteristics (for example wide dynamic range of sensors able to detect large, middle sized and very small objects which can increase the resolution abilities) and to give more clear picture of the investigated objects.
3. The extension on the familiar topics and the development of new areas of exploration is another challenge to be considered.

As it was demonstrated there is a broad field to the Bulgarian Fractal research which needs more extension. It shows a potential which could be used for development in future and to help the younger scientists to use this useful tool for variety of investigations going to the view from another unique side then of widespread usual techniques of research.

In this review the main aim was to present the development and the recent level of the fractal research in Bulgaria. It has not a target to give the full picture, but to synthesize the most important achievements in this field of knowledge. The examples presented, the bibliography (as Appendix), the gaps outlined and some future intensions could be a line of development in the near future of this topic of knowledge to the future generations in Bulgaria.

7. Conclusion

The presented Review gives the picture of the application and evolution of the fractals' investigations in Bulgaria covering different disciplines: Global Geodynamics, Seismotectonics and Seismic hazard assessment, Local and Regional morphostructures and relationships with the natural geophysical fields, Remote sensing and relationships of fractals in the Solar system and other fields of knowledge. Starting with the few results regarding the tsunami investigations and coastal interactions, extending its fields to the seismicity and seismic hazard analysis, touching other terrestrial bodies in the Solar system, all these research fields show the effective use of the fractal analysis as a powerful tool and its extension to many useful applications. The short history of this topic evolution gives the basis to assess the gaps and suggests the possible challenges of the fractal analysis and its applications for near future expectations in Bulgaria. Recently there are teams working on variety of topics, but the need of educated younger generation is an essential requirement for the successful development of this issue.

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