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Entire Functions Sharing a Second order Polynomial with its Derivatives

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Abstract: We prove a uniqueness theorem for an entire function, which share a function with their first and second order derivatives. We improve some existing results.
Keywords: Entire function, Polynomial, Uniqueness

1 Introduction, Definitions and Results

Let f be a non-constant meromorphic function in the open complex plane $\mathbb{C}$. We denote by $T(r, f)$ the Nevanlinna characteristic function of f and by $S(r, f)$ any quantity satisfying $S(r, f) = o\{T(r, f)\}$ as $r \to \infty$ except possibly a set of finite linear measure.

Let f and g be two non-constant meromorphic functions and let a be a complex number. We denote by $E(a; f)$ the set of a-points of f, where each point is counted according its multiplicity. We denote by $\overline{E}(a; f)$ the reduced form of $E(a; f)$. We say that f and g share a CM, provided that $E(a; f) = E(a; g)$, and we say that f and g share a IM, provided that $\overline{E}(a; f) = \overline{E}(a; g)$. In addition, we say that f and g share ∞ CM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 CM, and we say that f and g share ∞ IM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 IM.

For standard definitions and notations of the value distribution theory we refer the readers to [2]. However we require the following definitions.

Definition 1.1 A meromorphic function $a = a(z)$ is called a small function of f if $T(r, a) = S(r, f)$.

Definition 1.2 Let f and g be two non-constant meromorphic functions defined in $\mathbb{C}$. For $a, b \in \mathbb{C} \cup \{\infty\}$ we denote by $N(r, a; f \mid g \neq b)(\overline{N}(r, a; f \mid g \neq b)$ the counting function (reduced counting function) of those a-points of f which are not the b-points of g.

Definition 1.3 *Let f and g be two non-constant meromorphic functions defined in* $\mathbb{C}$. *For* $a, b \in \mathbb{C} \cup \{\infty\}$ *we denote by* $N(r, a; f \mid g = b)(\overline{N}(r, a; f \mid g = b)$ *the counting function (reduced counting function) of those a-points of f which are the b-points of g.*

In 1977 L.A.Rubel and C.C.Yang [7] first investigated the uniqueness of entire function sharing certain values with their derivatives. They proved the following result.

Theorem A *[7] Let* f *be a nonconstant entire function. If* $E(a; f) = E(a; f^{(1)})$ *and* $E(b; f) = E(b; f^{(1)})$ *for two distinct finite complex numbers* a *and* b *then* $f \equiv f^{(1)}$.

In 1979 E.Mues and N.Steinmetz [6] improved theorem A in the following manner.

Theorem B *[6] Let* a *and* b *be two distinct finite complex numbers and* f *be a nonconstant entire function. If* $\overline{E}(a; f) = \overline{E}(a; f^{(1)})$ *and* $\overline{E}(b; f) = \overline{E}(b; f^{(1)})$*, then* $f \equiv f^{(1)}$.

In 1986 Jank,Mues and Volkman [3] considered the problem of sharing a single value by the derivatives of an entire function. Their result may be stated as follows.

Theorem C *[3] Let f be a non-constant entire function and* $a(\neq 0)$ *be a finite complex number. If* $\overline{E}(a; f) = \overline{E}(a; f^{(1)})$ *and* $\overline{E}(a; f) \subset \overline{E}(a; f^{(2)})$, *then* $f \equiv f^{(1)}$.

In 2002 Chang and Fang [1] extended Theorem C and proved the following result.

Theorem D *[1] Let f be a non-constant entire function. If* $\overline{E}(z; f) = \overline{E}(z; f^{(1)})$ *and* $\overline{E}(z; f^{(1)}) \subset \overline{E}(z; f^{(2)})$, *then* $f \equiv f^{(1)}$.

In this paper, we will improve Theorem D by increasing the power of the sharing function z as well as relaxing the condition by considering one sided inclusion $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$ instead of $\overline{E}(z; f) = \overline{E}(z; f^{(1)})$ in Theorem D .

We give an example below to show that the Theorem 1.1 is not true if we consider $a(z) = z^2$ that is the Theorem 1.1 is not true for general second degree polynomial $a(z)$ .So $a(z) = z^2 + 1$ is necessary in Theorem 1.1.

Example 1.1 *Let* $f(z) = 2z^2 - 4z + 4$ *and* $a(z) = z^2$, *then* $f(z) - a(z) = z^2 - 4z + 4 = (z-2)^2$ *and* $f^{(1)}(z) - a(z) = 4z - 4 - z^2 = -(z-2)^2$ *and* $f^{(2)}(z) - a(z) = 4 - z^2 = (2-z)(2+z)$, *which means* $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$ *and* $\overline{E}(a; f^{(1)}) \subset \overline{E}(a; f^{(2)})$, *but* $f \neq Aexp\{z\}$ *or* $f \neq (z^2+1) + (z^2 - 4z + 5)exp\{\frac{z}{2+Bi}\}$, *where A is a non-zero constant and* $B^2 = 1$.

We now state the main result of the paper.

Theorem 1.1 *Let f be a non-constant entire function and* $a(z) = z^2 + 1$. *If* $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$ *and* $\overline{E}(a; f^{(1)}) \subset \overline{E}(a; f^{(2)})$, *then either* $f = Aexp\{z\}$ *or* $f = (z^2+1) + (z^2 - 4z + 5)exp\{\frac{z}{2+Bi}\}$ *where A is a non-zero constant and* $B^2 = 1$.

Corollary 1.1 *If in Theorem 1.1 we assume* $\overline{E}(a; f) = \overline{E}(a; f^{(1)})$ *, then* $f = Aexp\{z\}$, *where* $A(\neq 0)$ *is a constant.*

2 Lemmas

In this section we present a very important lemma which helps us to prove the theorem.

Lemma 2.1 *[4] Let* f *be a transcendental entire function and* $a = a(z)(\not\equiv 0, \infty)$ *be a non-constant small function of f such that* $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$ *and* $\overline{E}(a; f^{(1)}) \subset \overline{E}(a; f^{(2)})$. *Then* $f = Aexp\{z\}$ *if and only if* $m(r, \frac{1}{f-a}) = S(r, f)$, *where A is a non-zero constant.*

3 Proof of the theorem

Proof of Thorem \reft1. First we suppose that f is a transcendental entire function.
Let

$$\psi = \frac{(a-a^{(1)})(f^{(2)}-a^{(2)})-(a-a^{(2)})(f^{(1)}-a^{(1)})}{(f-a)} \tag{1}$$

where $a = z^2+1$.

If $\psi \equiv 0$. Then

$$\frac{f^{(2)}-2}{f^{(1)}-2z} = \frac{z^2-1}{z^2-2z+1}$$

i.e.,

$$\frac{f^{(2)}-2}{f^{(1)}-2z} = 1+\frac{2z-2}{z^2-2z+1}$$

This gives on integration

$$\log(f^{(1)}-2z) = z+\log(z^2-2z+1)+\log A \tag{2}$$

i.e.,

$$f^{(1)} = 2z + A(z^2-2z+1)exp\{z\}$$

i.e.,

$$f = z^2 + A(z^2-4z+5)exp\{z\}+B, \tag{3}$$

and

$$f^{(2)} = 2 + A(z^2-1)exp\{z\}.$$

Where $A(\neq 0)$ and B are constants.

Let z_0 be a solution of $f(z)-a(z)=0$.

Then

$$f(z_0)-(z_0^2+1) = A(z_0^2-4z_0+5)exp\{z_0\}+B-1 = 0, \tag{4}$$

$$f^{(1)}(z_0)-(z_0^2+1) = A(z_0^2-2z_0+1)exp\{z_0\}-(z_0^2-2z_0+1) = 0, \tag{5}$$

and

$$f^{(2)}(z_0)-(z_0^2+1) = A(z_0^2-1)exp\{z_0\}+(1-z_0^2) = 0, \tag{6}$$

From (5) we get

$$(z_0^2-2z_0+1)(Aexp\{z_0\}-1) = 0$$

i.e., $Aexp\{z_0\}=1$ or $z_0=1$.

If $z_0=1$ then the equation (2) does not exist, so $z_0 \neq 1$.

If $Aexp\{z_0\}=1$ then from the equation (4) we get,

$$z_0^2-4z_0+4+B = 0$$

i.e., $z_0 = 2\pm\sqrt{B}i$. That is $f(z)-(z^2+1)=0$ has two solutions $z_0 = 2\pm\sqrt{B}i$. Also from (3) $f(z)-(z^2+B)=0$ implies $Aexp\{z\}(z^2-4z+5)=0$,since $Aexp\{z\}\neq 0$ then $z=2\pm i$. Hence $f(z)-(z^2+B)=0$ has two solutions $z=2\pm i$. We conclude that $\sqrt{B}=\pm 1$ i.e., $B=1$ and $A = exp\{\frac{1}{2\pm i}\}$.

Putting the value of A and B in (3) we get,

$$f(z) = z^2 + 1 + (z^2 - 4z + 5)exp\{\frac{z}{2 \pm i}\}.$$

Now we suppose that $\psi \not\equiv 0$ .Then

$$f - a \equiv \frac{1}{\psi}[(a - a^{(1)})(f^{(2)} - a^{(2)}) - (a - a^{(2)})(f^{(1)} - a^{(1)})] \qquad (7)$$

where $a = z^2 + 1$.

And so

$$[1 + (\frac{1}{\psi})'(a - a^{(2)}) + \frac{a^{(1)}}{\psi}](f^{(1)} - a) \equiv (a^{(1)} - a)[1 + (\frac{1}{\psi})'(a - a^{(1)}) + \frac{2}{\psi}(a^{(1)} - a^{(2)})]$$
$$+ (a^{(1)} - a)[\frac{1}{\psi} - (\frac{1}{\psi})'](f^{(2)} - a^{(1)}) - (a^{(1)} - a)\frac{f^{(3)} - a^{(2)}}{\psi} \qquad (8)$$

Let

$$\Delta = 1 + (\frac{1}{\psi})'(a - a^{(1)}) + \frac{2}{\psi}(a^{(1)} - a^{(2)}) \equiv 0$$

i.e.,

$$1 + (\frac{1}{\psi})'(z^2 - 2z + 1) + \frac{2}{\psi}(2z - 2) \equiv 0 \qquad (9)$$

i.e.,

$$\psi^2 + 4(z-1)\psi \equiv \psi'(z^2 - 2z + 1) \qquad (10)$$

We claim that ψ is not transcendental.
Indeed, if ψ is transcendental,then from (10) we get

$$T(r,\psi) = m(r,\psi) + N(r,\psi)$$
$$\le m(r, \frac{\psi'}{\psi}) + O(\log r)$$
$$= S(r,\psi).$$

Thus we get a contradiction: $T(r,\psi) = S(r,\psi)$.
Hence ψ is a rational function. Solving the differential equation (10) we get,

$\psi = \frac{-3(z-1)^4}{(z-1)^3 + 3k} = az + b(say)$ where $a(\ne 0)$,b and k are constants.

Put $\psi = az + b$ in (10) and equating the coefficients of z^2, z and constant term both the sides we get, $a^2 + 4a = a$ i.e., $a = 0$ or $a = -3$ but $a \ne 0$ so $a = -3$ and $b = 3$. Hence $\psi = -3(z-1)$.
If we put $\psi = -3(z-1)$ in (1) we get,

$$-3(z-1)(f - z^2 - 1) = (z^2 - 2z + 1)(f^{(2)} - 2) - (z^2 - 1)(f^{(1)} - 2z)$$

i.e.,

$$(z-1)\{(z-1)(f^{(2)} - 2) - (z+1)(f^{(1)} - 2z) + 3(f - z^2 - 1)\} = 0$$

i.e.,

$$z = 1 \, or \, (z-1)(f^{(2)} - 2) - (z+1)(f^{(1)} - 2z) + 3(f - z^2 - 1) = 0 \qquad (11)$$

If $z = 1$ then from (9) we get $1 \equiv 0$, which is a contradiction.
Now differentiating thrice of the equation (11) we get

$$\frac{f^{(5)}}{f^{(4)}} = \frac{z-2}{z-1} = 1 - \frac{1}{z-1}$$

On integration we obtain

$$f^{(4)} = \frac{c.exp\{z\}}{z-1},$$

where $c \neq 0$ is a constant.This is not possible because f is an entire function.
Therefore $\Delta \not\equiv 0$ and so from (??) we obtain

$$\frac{1}{f^{(1)} - a} \equiv \frac{1 + (\frac{1}{\psi})'(a - a^{(2)}) + \frac{a^{(1)}}{\psi}}{(a^{(1)} - a)\Delta} - \frac{(\frac{1}{\psi} - (\frac{1}{\psi})')(f^{(2)} - a^{(1)})}{\Delta(f^{(1)} - a)} + \frac{1}{\Delta\psi}.\frac{f^{(3)} - a^{(2)}}{f^{(1)} - a}.$$

Hence

$$m(r, \frac{1}{f^{(1)} - a}) = S(r, f) \tag{12}$$

because $T(r,\psi) = S(r, f)$ and f is transcendental.
By the hypotheses we see that $z = 1\, and\, -1$ are only the possible multiple zero of $f^{(1)} - a$.
So,

$$N(r, a; f^{(1)} \mid f \neq a) \leq N(r,0;\psi) + O(\log r) = S(r, f).$$

Also since $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$ then

$$N(r, a; f^{(1)}) = N(r, a; f) + N(r, a; f^{(1)} \mid f \neq a) + O(\log r) = N(r, a; f) + S(r, f). \tag{13}$$

From (7) we get,

$$f = a + \frac{f^{(1)} - a^{(1)}}{\psi}[(a - a^{(1)}).\frac{f^{(2)} - a^{(2)}}{f^{(1)} - a^{(1)}} - (a - a^{(2)})]$$

Hence,

$$\begin{aligned} m(r, f) &\leq m(r, f^{(1)} - a^{(1)}) + S(r, f) \\ &\leq m(r, f^{(1)}) + S(r, f) \\ &= T(r, f^{(1)}) + S(r, f). \end{aligned}$$

Since f is an entire function we get,

$$T(r, f) = m(r, f) \leq T(r, f^{(1)}) + S(r, f) \tag{14}$$

Also,

$$T(r, f^{(1)}) = m(r, f^{(1)}) \leq m(r, f) + m(r, \frac{f^{(1)}}{f}) = T(r, f) + S(r, f) \tag{15}$$

Therefore

$$T(r, f) = T(r, f^{(1)}) + S(r, f). \tag{16}$$

From (12),(13) and (16) we get

$$m(r,\frac{1}{f-a}) = T(r,f) - N(r,\frac{1}{f-a}) + S(r,f)$$
$$= T(r,f^{(1)}) - N(r,\frac{1}{f-a}) + S(r,f)$$
$$= N(r,\frac{1}{f^{(1)}-a}) - N(r,\frac{1}{f-a}) + S(r,f)$$
$$= S(r,f).$$

Therefore by Lemma 2.1 we get $f = Aexp\{z\}$.

Now we prove that f can not be a polynomial. We suppose that f is a polynomial and consider the following cases.

Case 1. Let $f = Az + B$, where $A(\neq 0)$ and B are constants, and $a(z) = z^2 + 1$ then if z_0 is a root of $f(z) - (z^2+1) = 0$, then by hypotheses z_0 is also a root of $f^{(1)}(z) - (z^2+1) = 0$ and $f^{(2)}(z) - (z^2+1) = 0$. Hence $A - (z_0^2+1) = 0$ and $0 - (z_0^2+1) = 0$ i.e., $A = z_0^2 + 1 = 0$, which is a contradiction.

Case 2. Let $f = Az^2 + Bz + C$, where $A(\neq 0)$, B and C are constants. If $f(z) - a(z) = 0$ has two distinct roots z_1 and z_2 , then by hypotheses z_1 and z_2 are also roots of $f^{(1)}(z) - (z^2+1) = 0$ and $f^{(2)}(z) - (z^2+1) = 0$. That is z_1 and z_2 are roots of $2Az + B - (z^2+1) = 0$ and so $z_1 + z_2 = 2A$. Also z_1 and z_2 are roots of $2A - (z^2+1) = 0$ and so $z_1 + z_2 = 0$. Hence $2A = 0$ i.e., $A = 0$, a contradiction.

So $f(z) - a(z) = 0$ has only one double root z_0.Then by hypotheses z_0 is also a root of $f^{(1)}(z) - a(z) = 0$ and $f^{(2)}(z) - a(z) = 0$. So,

$$Az_0^2 + Bz_0 + C - z_0^2 - 1 = 0 \quad (17)$$

$$2Az_0 + B - z_0^2 - 1 = 0 \quad (18)$$

$$2Az_0 + B - 2z_0 = 0 \quad (19)$$

$$2A - z_0^2 - 1 = 0 \quad (20)$$

Solving these four equations we obtain $A = 1, B = 0$ and $C = 1$. So, $f(z) = z^2 + 1$ i.e., $f(z) \equiv a(z)$. Since $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$, we arrive at a contradiction.

Case 3. Let f be a polynomial of degree 3. Suppose $f = Az^3 + Bz^2 + Cz + D$, where $A(\neq 0)$, B,C and D are constants.

Subcase 3.1. First we suppose that $f(z) - a(z) = 0$ has three distinct roots. Since $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$ so these three roots are also the roots of the equation $f^{(1)}(z) - a(z) = 0$ i.e., of the equation $3Az^2 + 2Bz + C - (z^2+1) = 0$, which is possible when $f^{(1)}(z) \equiv a(z)$. Since

$\overline{E}(a, f^{(1)}) \subset \overline{E}(a, f^{(2)})$, we arrive at a contradiction.

Subcase 3.2. Now we suppose that $f(z)-a(z)=0$ has one double root and one simple root. Let z_1 be a double root and z_2 be a simple root of the equation $f(z)-a(z)=0$. Then by hypotheses,

$$Az_1^3 + Bz_1^2 + Cz_1 + D - z_1^2 - 1 = 0 \tag{21}$$

$$3Az_1^2 + 2Bz_1 + C - z_1^2 - 1 = 0 \tag{22}$$

$$6Az_1 + 2B - z_1^2 - 1 = 0 \tag{23}$$

$$3Az_1^2 + 2Bz_1 + C - 2z_1 = 0 \tag{24}$$

Solving these four equations we obtain, $B = 1-3A$, $C = 3A$ and $D = 1-A$. And so $f(z) = A(z-1)^3 + (z^2+1)$. Hence the equation $f(z)-a(z) = A(z-1)^3$ has only one root of multiplicity three which contradicts our assumption that $f(z)-a(z)=0$ has one double root and one simple root.

Subcase 3.3. Now we suppose that $f(z)-a(z)=0$ has only one root of multiplicity three. Let z_1 be the root of multiplicity three of the equation $f(z)-a(z)=0$. Then by hypotheses, we obtain the equations (21)-(24) and the equation

$$6Az_1 + 2B - 2 = 0 \tag{25}$$

Solving the equations (21)-(24) we obtain, $f(z)-a(z) = A(z-1)^3$. But from (23) and (25) we get $z_1^2 - 1 = 0$ i.e., $z_1 = 1$ and -1, so -1 also a root of the equation $f^{(2)}(z)-a(z)=0$ i.e., of the equation $6A(z-1)+2-z^2-1=0$, if we put $z=-1$ of this equation we get $A=0$, which is a contradiction.

Case 4. Let f be a polynomial of degree $d(\geq 4)$. If $z_1, z_2, ..., z_n$ are the roots of the equation $f(z)-a(z)=0$. Then we have

$$f(z) = (z^2+1) + A(z-z_1)^{\alpha_1}(z-z_2)^{\alpha_2}...(z-z_n)^{\alpha_n} \tag{26}$$

$$f^{(1)}(z) = (z^2+1) + B(z-z_1)^{\beta_1}(z-z_2)^{\beta_2}...(z-z_n)^{\beta_n}P(z) \tag{27}$$

$$f^{(2)}(z) = (z^2+1) + C(z-z_1)^{\gamma_1}(z-z_2)^{\gamma_2}...(z-z_n)^{\gamma_n}P(z)Q(z) \tag{28}$$

where $P(z), Q(z)$ are polynomials and A,B,C are three non-zero constant, and $\{\alpha_j\}$, $\{\beta_j\}$, $\{\gamma_j\}$ (j=1,2,...n) are positive integers satisfying

$$\begin{aligned} &\alpha_1 + \alpha_2 + ... + \alpha_n = d, \beta_1 + \beta_2 + ... + \beta_n + degP = d-1, \\ &and\ \gamma_1 + \gamma_2 + ... + \gamma_n + degP + degQ = d-2. \end{aligned} \tag{29}$$

Differentiating equation (26) and equate with (27) we get,

$$2z + A\sum_{i=1}^{n}\alpha_i(z-z_i)^{\alpha_i - 1}\prod_{j\neq i}(z-z_j)^{\alpha_j} \equiv (z^2+1) + B\prod_{j=i}^{n}(z-z_j)^{\beta_j}P(z) \tag{30}$$

If $\alpha_j \geq 2$. Then by (30) we get $z_j = 1$. With out loss of generality, we assume that $j=1$. Then by (26),(27) and (??) we obtain,

$$f(z) = (z^2+1) + A(z-1)^{\alpha_1}(z-z_2)...(z-z_n) \tag{31}$$

$$f^{(1)}(z) = (z^2+1) + B(z-1)^{\alpha_1 - 1}(z-z_2)...(z-z_n) \tag{32}$$

Differentiating twice of the equation (26) and equating with the equation (28) we get ,

$$2 + A\sum_{i=1}^{n}\alpha_i(\alpha_i - 1)(z-z_i)^{\alpha_i - 2}\prod_{j\neq i}(z-z_j)^{\alpha_j} + 2A\sum_{i,j=1}^{n}\alpha_i\alpha_j(z-z_i)^{\alpha_i - 1}(z-z_j)^{\alpha_j - 1}$$

$$\prod_{k\neq i,j}(z-z_k)^{\alpha_k} = (z^2+1) + C\prod_{i=i}^{n}(z-z_i)^{\gamma_i}P(z)Q(z) \tag{33}$$

If any $\alpha_i \geq 3$ then from (??) we obtain, $2 = z_i^2 + 1$ i.e., $z_i = 1\, and\, -1$. With out loss of generality we put $z_1 = -1$ and $\alpha_1 \geq 3$. Then -1 is a root of $f(z) - (z^2+1) = 0$ but $f^{(1)}(-1) = -2$ i.e., -1 is not a root of the equation $f^{(1)}(z) - (z^2+1) = 0$. Thus we see that $-1 \in \overline{E}(a, f)$ but $-1 \notin \overline{E}(a, f^{(1)})$ which contradicts the hypothesis $\overline{E}(a; f) \subset \overline{E}(a; f^{(1)})$. Thus any α_i not greater or equal to 3. Thus $\alpha_1 = 2$.

Hence by (32) we get

$$f^{(1)}(z) = (z^2+1) + B(z-1)(z-z_2)...(z-z_n)$$

and

$$f^{(2)}(z) = (z^2+1) + C(z-1)(z-z_2)...(z-z_n)$$

Thus we arrive at a contradiction: $degf^{(1)} = degf^{(2)}$. This proves the theorem.

Proof of Corollary 1.1.. If $f = (z^2+1) + (z^2 - 4z + 5)exp\{\frac{z}{2+Bi}\},$ where $B^2 = 1$. Then $f^{(1)} = 2z + (2z-4)exp\{\frac{z}{2+Bi}\} + \frac{(z^2-4z+5)}{2+Bi}exp\{\frac{z}{2+Bi}\}$ we clearly see that $\overline{E}(a; f)$ contains only two points but $\overline{E}(a; f^{(1)})$ contains infinitely many points. This is a contradiction of the hypothesis $\overline{E}(a; f) = \overline{E}(a; f^{(1)})$. Hence by Theorem 1.1 we get $f = Aexp\{z\}$.

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