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Wave Solutions of The Benjamin-Bona-Mahony's Equation In Weighted Sobolev Spaces

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ABSTRACT:*We explore the*conformable fractional BBM equation. *The question of the center of mass of the wave solution is analyzed with the existence of solution for the Benjamin-Bona- Mahony' s equation (BBM) in a weighted Sobolev space. Similar results of the center of energy in DNA breathing is observed using Klein Gordon equations and Floquet theory.*

KEYWORDS:*soliton,,center of mass, DNA breathing, Klein Gordon, Floquet theory*

I. INTRODUCTION

Theconceptual terminology solitons was introduced by Zabusky and Kruskal [16] to refer to travelling waves having a stability property in the interaction . Another property is the center of mass of this soliton. Generally we can prove the center of mass of the soliton is asymptotically located on a line. Similar results we can observe in the center of energy of the breather mobile in DNA using Klein-Gordon equations [14]. J. Labadin use Navier Stokes for the study of the arterial Blood flow. The blood pressure refers is the force per unit area that blood exerts on the walls of blood vessels. We know that the curve blood pressure candetect of cardiac arrhythmia -irregular heartbeat,estimate cardiac output, estimate hypovolemia (a decreased volume of circulating blood in the body). Also, we have some problems in monitoring respiratory variation (which in turn may be related to fluid responsiveness in ventilated patients with circulatory failure) and detecting attenuation of the peripheral pulse waveform depending on different base pathologies such as sepsis or severe respiratory distress, such attenuation damping may be considered an important measure for the assessment of mmicrocirculation and tissue perfusion.We consider the initial value problem for the regularized long-wave equation of Peregrine[12] and Benjamin-Bona-Mahony:

$$\partial_t u + \partial_x u + u\partial_x u - \partial_x^2 \partial_t u = 0 \quad (1)$$

For all $x \in \mathbb{R}$, $t > 0$

The general form is the conformable fractional BBM equation:

$$u^{\alpha}{}_{t} + \varepsilon u_x + \beta u u_x + \mu u^{\alpha}_{xxt} = 0 \quad (1')$$

where ε,β,μ are ´parameters and 0<α ≤1 denote the conformable fractional derivative. We discuss the global existence for the fractional Benjamin-Bona Mahony's equation in a weighted Sobolev space.The reference [1] give the global solution for the equation (1) for when the initial data is given in a suitable weighted Sobolev space. In this case the weight is of polynomial type [1]. Following the same methodology of reference [1] we can propose the hypothesis of the global existence of the solution for the problem (1') and also the direction of the center of the mass of the solitons is precisely a line. Similar results we have obtained for the DNA breathing [14]. A. Korkmaz [15]obtain solutions for the problem (1') using the wave transformation:

u=u(ξ) , ξ=x-(c/α) t^{α} (1'') .

In the reference [14] we have discussed the existence of mobile breather for the DNA breathing problem and analyzed the center of energy. The velocity of the mobile breather in [14] is analyzed. Generally the existence of the mobile breather depends of the harmonic bifurcation and analyzed using the Floquet Theory.

II. METHODOLOGY

BBM problem when the weight is the polynomial type: The Benjamin-Bona-Mahony's equation (BBM) is given by:

$$\partial_t u + \partial_x u + u\partial_x u - \partial_x^2 \partial_t u = h(x)\,, x \in \mathbb{R}, t > 0 \tag{1}$$

Notation and preliminaries

Let X a Hilbert space and $m \in \mathbb{N}$, $T > 0$ we denote by $C^m([0,T];X)$ the space of vector. Valued functions $u:[0,T] \to X$ m-times continuously differentiable in $[0,T]$ with the usual norm:

$$\mu(x) = (1 + |x|^2)^{1/2}$$

For every $s \in \mathbb{R}$, the Sobolev spaces $H^s(\mathbb{R})$ are introduced as usual with the norm $\|u\|_s = |\mu^s \hat{u}|_2$ where the $|.|$ denotes the standard norm of $L^2(\mathbb{R})$ and $\hat{u}$ is the Fourier transform of u . We also denote by

$$\Lambda^s = (I - \partial_x^2)^{s/2}$$

The Bessel potential of order s.
If $r \in \mathbb{R}$ we denote by M_r the multiplication operator with the function $\mu^r(x)$. While if $r, s \in \mathbb{R}$ we denote by $H_r^s(\mathbb{R}) = H^s(\mathbb{R}, \mu^r(x)dx)$ the completion of Schwarz space $\mathcal{S}(\mathbb{R})$ in the norm

$$|u|_{r,s} = |M_r \Lambda^s u|_2$$

$H_r^s(\mathbb{R})$ is a Hilbert space with the inner product
$(u,v)_{r,s} = (M_r \Lambda^s u, M_r \Lambda^s v)_2$ and its dual is defined by $\left(H_r^s(\mathbb{R})\right)' = H_{-r}^{-s}(\mathbb{R})$. When $r = 0$, the space H_0^s coincide with the usual norm Sobolev space. The case $r = s = 0$ we put
$H_0^0(\mathbb{R}) = L^2(\mathbb{R})$.
We introduce the Hilbert space $\mathcal{S}_r^s(\mathbb{R})$ defined by

$$\mathcal{S}_r^s(\mathbb{R}) = H_r^0(\mathbb{R}) \cap H_0^s(\mathbb{R})$$

For every $r, s \in \mathbb{R}$ with the inner product

$$(((u,v)))_{r,s} = (u,v)_{r,0} + (u,v)_{0,s}$$

We give some properties
i)Forevey $r_1, s_1, r_2, s_2 \in \mathbb{R}$ and $\theta \in (0,1)$

$$\left[H_{r_1}^{s_1}, H_{r_2}^{s_2}\right]_\theta = H_{(1-\theta)r_1+\theta r_2}^{(1-\theta)s_1+\theta s_2}$$

Where $[X,Y]_\theta$ is the interpolation space of X and Y.
ii) For every $r, s \in \mathbb{R}$ there exists c_1, c_2 positive constants such that

$$c_1 |u|_{r,s} \le |\mathcal{F}^{-1}(M_s \mathcal{F}(M_r u))|_2 \le c_2 |u|_{r,s}$$

For every $s \in H_r^s(\mathbb{R})$
iii) For every $r, s \in \mathbb{R}$, $\mathcal{F}(H_r^s(\mathbb{R})) = H_s^r(\mathbb{R})$
iv) If $r_1 \ge r_2, s_1 \ge s_2$ then
$H_{r_1}^{s_1} \subset H_{r_2}^{s_2}$, $\mathcal{S}_{r_1}^{s_1} \subset \mathcal{S}_{r_2}^{s_2}$ with continuous injection.
v) For every $r, s \in \mathbb{R}$ and $\theta \in (0,1)$

$$\mathcal{S}_r^s(\mathbb{R}) \subset [H_r^0(\mathbb{R}), H_0^s(\mathbb{R})]_\theta = H_{(1-\theta)r}^{\theta s}$$

vi) If $r_1, s_1, r_2, s_2, r, s \in \mathbb{R}$ and $u \in H_{r_1}^{s_1}(\mathbb{R}), v \in H_{r_2}^{s_2}(\mathbb{R})$ then $uv \in H_r^s(\mathbb{R})$ whenever

$$s_1, s_2 \ge s\,, \qquad s_1 + s_2 - s \ge 1/2$$

Moreover there exists a constant $c > 0$ such that:

$$|uv|_{r,s} \le c|u|_{r_1,s_1} |u|_{r_2,s_2}$$

For some other details one find in (cf.[4],[14].

III. RESULT AND DISCUSSION

Mild solution

Theorem 1. Let $r \ge 0$ if $u_0 \in H^4(\mathbb{R}) \cap H_r^2(\mathbb{R})$
then the initial value problem

$$\begin{cases} \partial_t u + \partial_x u + u\partial_x u - \partial_x^2 \partial_t u = 0 \,, x \in \mathbb{R}, t > 0 \\ u(x,0) = u_0(x) \quad , \quad x \in \mathbb{R} \end{cases}$$

(2)

Has a unique solution

$$u \in C^0([0,\infty); H^4(\mathbb{R}) \cap H_r^2(\mathbb{R})) \cap C^1([0,\infty); H_r^2(\mathbb{R}))$$

Before proceeding to the proof, we establish some preliminary results. We know that (2) is equivalent to the following initial-value problem:

$$\begin{cases} \partial_t u = Bu + Bfu \quad , x \in \mathbb{R}, t > 0 \\ u(x,0) = u_0(x) \quad , \quad x \in \mathbb{R} \end{cases}$$

Where $B = -\Lambda^{-2}\partial_x$, $f(s) = s^2/2$

Lemma1. $B: H_r^2(\mathbb{R}) \to H_r^2(\mathbb{R})$is a linear bounded operator :

$|Bu|_{r,2} \leq c|u|_{r,2}$(for a review, see ref. 1).

Proposition1. (local existence)Given $u_0 \in H_r^2(\mathbb{R})$ there exists $T \in (0,\infty)$ and an unique mild solution u of (1) in $\mathbb{R} \times (0,T)$ with initial data u_0. Here a mild solution is an element of $C([0,T]; H_r^2(\mathbb{R}))$ satisfying

$$u(t) = e^{Bt}u_0 + \int_0^t e^{B(t-\sigma)} B\left(\frac{u(\sigma)^2}{2}\right) d\sigma$$

We use Lemma1 for concluding that B is the infinitesimal generator of a uniformly continuous semigroup of operators on $H_r^2(\mathbb{R})$. The conclusion follows using standard arguments given in Pazy[11].

Proof of Theorem 1

Note that Bf is differentiable, $u(x,t)$ is a classical solution of (1) on any $t-$interval where it exists. Furthermore

$$\partial_t u = Bu + Bfu \in C([0,T]; H_r^2(\mathbb{R}))$$

For every $s \geq 0$ the existence of a globalH^s-valued solution for (1) has already been proved (cf.[1],[2]). In addition, such solution belongs to $C^0([0,\infty); H^s(\mathbb{R}))$.

We claim that for every $T > 0$, there exists some constant$M_T > 0$ such that

$$\sup_{0\leq t\leq T} |u(.,t)|_{r,2} \leq M_T$$

In fact, for every $u_0 \in H_r^2(\mathbb{R})$ by the density of $\mathcal{S}_{2r}^4(\mathbb{R})$ in $H_r^2(\mathbb{R})$ (cf.[14])there exists

$(u_{0n}) \in \mathcal{S}_{2r}^4(\mathbb{R})$ such that

$u_{0n} \to u_0$ strong in $H_r^2(\mathbb{R})$

Now consider $u_n = u_n(x,t)$ be the solution of initial value problem.

$$\begin{cases} \partial_t u_n + \partial_x u_n + u\partial_x u_n - \partial_x^2 \partial_t u_n = 0 \,, x \in \mathbb{R}, t > 0 \\ u_n(x,0) = u_{0n}(x) \quad , \quad x \in \mathbb{R} \end{cases}$$

Then the solution u_n exists globally in $C([0,\infty); \mathcal{S}_{2r}^4(\mathbb{R}))$, (cf.[13]). In particular given any $T > 0$ there exists a constant M_T(independentof n) such that

$$||u_n||_{2r,4} \leq M_T$$

And (u_n) converges to the solution $u = u(x,t)$ of the problem (2). Since we have the embedding injection $\mathcal{S}^4_{2r}(\mathbb{R}) \subset H^2_r(\mathbb{R})$ it follows that

$$|u_n(.,t)|_{r,2} \leq C||u_n||_{2r,4} \leq M_T$$

Finally, we claim that

$$|u(.,t)|_{r,2} \leq \lim_{n\to\infty} inf |u_n(.,t)|_{r,2} \leq M_T$$

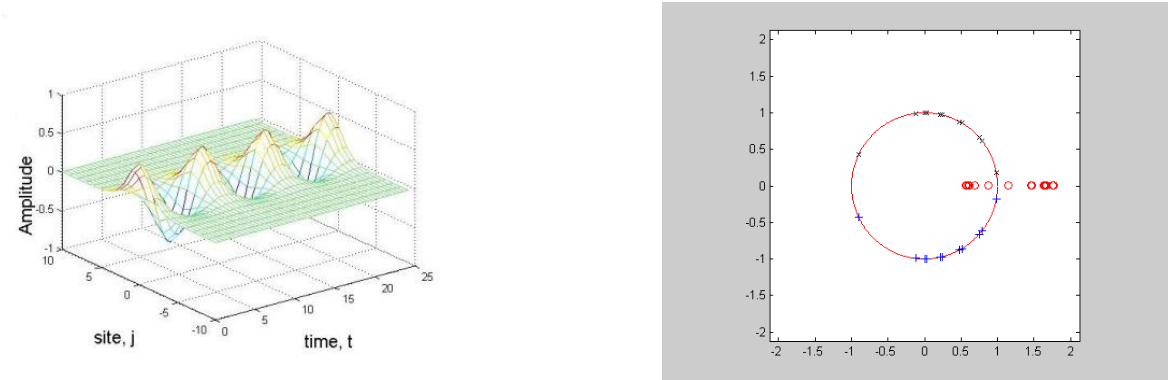
This completes the proof.

Remarks: Miller and Weinstein [9] proved an asymptotic stability of soliton for the BBM equation using exponential decay spaces.

Similar results for the DNA breathing: The Hamiltonian for DNA dynamics

$$H = \sum_{n=1}^{N} \frac{m}{2}\dot{u}_n^2 + \sum_{n=1}^{N} \frac{K}{2}(u_n - u_{n-1})^2 + \sum_{n=1}^{N} V_n(u_n) \quad (3)$$

where N = number of the pairs of bases; K = coupling constant; $velocity = \dot{u}$; and u_n = stretching of the hydrogen bonds = $(X_n - Y_n)/\sqrt{2}$; V is the Morse potential.



(a) **(b)**

Figure 1. (a) Stationary solution for the Hamiltonian; (b) The instability "harmonic bifurcation" with the evolution of the Fouquet multipliers with the parameters: *K*=0.004, wb=0.8 for the breather solution for the symmetric Morse potential (for a review, see ref. 17).

The center of energy of the breather mobile is given by

$$X_E = \sum_{n=1}^{N} nH_n^d \ / H \quad (4)$$

Where the density energy has the form

$$H_n^d = \frac{1}{2}\dot{u}_n^2 + \frac{K}{4}(u_n - u_{n-1})^2 + \frac{K}{4}(u_{n+1} - u_n)^2 + V_n(\sqrt{2}u_n) \quad (5)$$

The center of energy moves precisely on a line (for a review, see ref. 14).

IV. CONCLUSION

We have discuss the property of the solitons of the conformable fractional BBM equation (1').The center of energy of the DNA breathing moves precisely on a line.

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